

# MODELING OF SPIRAL INDUCTORS ON LOSSY SUBSTRATES FOR RFIC APPLICATIONS

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*Abstract*—A modeling technique for planar microstrip spiral inductors in the presence of lossy media, such as in CMOS and bipolar technologies, is presented. The new model for spiral inductors incorporates the frequency dependent substrate effects via analysis of the ‘skin-effect’ mode of the microstrip structure.

## I. Introduction

RFIC designs in lossy media incorporating passive structures such as microstrip spiral inductors require modeling techniques which properly address the loss mechanisms involved, particularly those losses due to semiconducting substrate effects. Spirals fabricated in Si-based circuits often yield poor quality factors due to high-resistivity metallization and lossy substrates. In addition, substrate currents can adversely affect the inductance of a spiral for certain frequencies and substrate conductivities.

Numerous design techniques for spiral inductors attempting to improve performance have been proposed and tested, including the use of multiple metallization layers in parallel [1], high resistivity substrates [2], as well as air suspended spirals [3]. While the properties of microstrip lines above lossy media have been studied extensively, [4],[5], current modeling techniques for spirals fabricated on lossy substrates do not properly incorporate all substrate effects, particularly substrate currents, which can have a dominating effect for certain frequencies, substrate conductivities, and geometries. Hence, a general analysis of the sub-

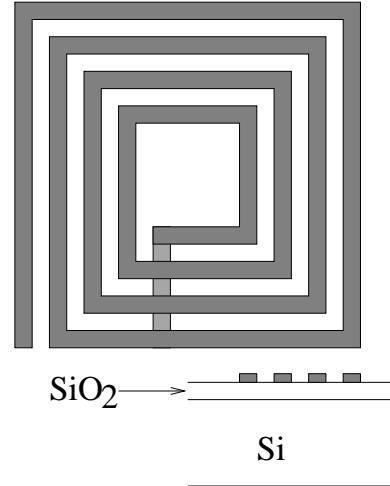


Fig. 1. Example of a Si-SiO<sub>2</sub>-based spiral structure

strate effects and development of a modeling technique is necessary for both simple single-level spirals (SLS) as well as more complex structures such as the designs described in [1]-[3].

This paper presents a new modeling technique for spirals mounted over moderately conductive substrates (e.g.  $\sigma = 10^4$  S/m), such as those used in CMOS RFICs. The analysis and simulations are performed for cases of SLS's in oxide over bulk Si substrates. An example structure is shown in Fig. 1.

## II. General Modeling Techniques

For a general spiral structure above a low-loss substrate, the modeling methodologies are well known and generally consist of lumped element

equivalent circuits [6]. Partial inductances and capacitances may be determined from closed form equations [7],[8] and, when included with the strip resistance, form the basis of an equivalent circuit representation for a low-loss structure. Additional losses due to conductor metallization skin depth and imperfect dielectrics may also be easily incorporated.

Likewise, equivalent circuit models for spirals based in Si-SiO<sub>2</sub> systems have been developed for 'high'-resistivity substrates (e.g. 10 S/m) [1],[9]. Typically, the substrate losses are addressed via inclusion of shunt conductances to represent the Silicon substrate and associated transverse (shunt) currents. This proves to be sufficient for conductivities on the order of 10 S/m or less, and frequencies in the low Gigahertz range. However, when the product of conductivity and frequency ( $\sigma\omega$ ) becomes significantly large, the skin depth,  $\delta = 1/\sqrt{\pi\mu\sigma f}$ , of the semiconducting substrate is small, and the existing equivalent circuit spiral models are not adequate. Thus, additional effects must also be addressed and included in an accurate model for these spiral inductors with higher conductivity substrates and frequencies of operation.

### III. Microstrip in Si-SiO<sub>2</sub>

First, we consider propagation characteristics of an Si-SiO<sub>2</sub> microstrip configuration as a basic element of the planar spiral inductor. The planar conductor is above a double layer substrate consisting of oxide over bulk Silicon. As described in the literature [4], three fundamental modes, namely, slow-wave, skin-effect, and quasi-TEM, will propagate in such a system, each having a certain frequency range for a given set of material parameters. Of particular interest for the case of the microstrip spirals in CMOS technology investigated in this paper is the skin-effect mode, as this is the propagating mode near the first resonance.

The skin-effect mode begins to propagate when the skin depth  $\delta$  is on the same order as or less than the Silicon substrate height. In other words, with increasing frequency, the semiconducting substrate begins to behave as a lossy con-

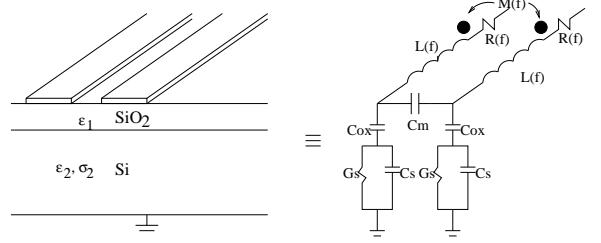


Fig. 2. Equivalent circuit for Si-SiO<sub>2</sub> microstrip coupled lines in skin effect mode

ductor wall and the longitudinal currents, which lower inductance per unit length, are closer to the Silicon/oxide interface. Application of Wheeler's method (incremental inductance rule) [10] to this case yields an inductance and associated loss that are both frequency dependent. An equivalent circuit model for a Si-SiO<sub>2</sub> coupled microstrip line configuration is shown in Fig. 2. The capacitances and shunt conductances are determined via a quasi-static Spectral Domain Analysis (SDA) [11] with complex dielectric constants. Distributed inductances and resistances are calculated using a modified Partial Element Equivalent Circuit (PEEC) [8] methodology including a frequency dependence to account for the skin effect mode. The frequency dependent series resistances incorporate the effects of finite skin depth in the metallization as well as loss in the semiconducting substrate.

### IV. Spiral Model for High $\sigma$ Si

The coupled line model described in the previous section forms the basis for an equivalent circuit model for a complete spiral. Such a distributed model is constructed by subdividing the structure and computing an equivalent circuit for each straight section (or leg) and including all capacitive and inductive coupling terms. This distributed model may become unnecessarily complex for a large number of discretizations, hence, it is desirable to have a means for model reduction. This may be accomplished via construction of an  $n$ -section ladder network, as shown in Fig. 3, where  $n$  is the number of turns. The reduced model gives virtually the same performance in the frequency range of interest, which is usually from dc to the first resonance. The order,  $n$ , of this par-

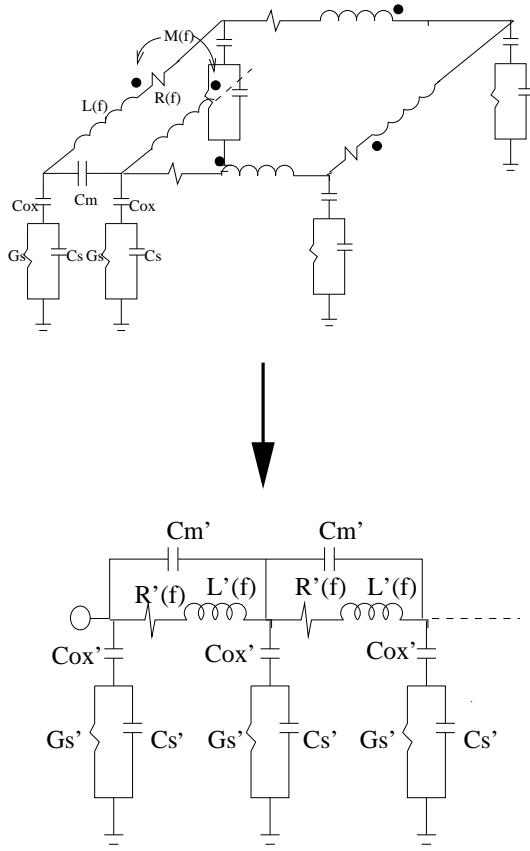


Fig. 3. Synthesis of a ladder network from the distributed model for turns of a spiral inductor

ticular circuit is, in general, high enough to sufficiently approximate the response of the original distributed model.

## V. Simulation/Measurement Results

As an example of a structure with relatively low substrate conductivity, a 5 nH spiral from [9] with  $\sigma_{Si} = 10 \text{ S/m}$  is simulated using a model which does not address the skin effect mode. The results are adequate for this particular structure, as can be observed in Fig. 4.

Next, we consider a 10 nH spiral fabricated in CMOS technology with  $\sigma_{Si} = 10^4 \text{ S/m}$ . Fig. 5 shows a comparison of three models with measured data. The proposed distributed model and corresponding ladder network (Fig. 3) properly account for the skin effect mode, in contrast to the model that only includes shunt conductances to represent the substrate losses. It is clear that the

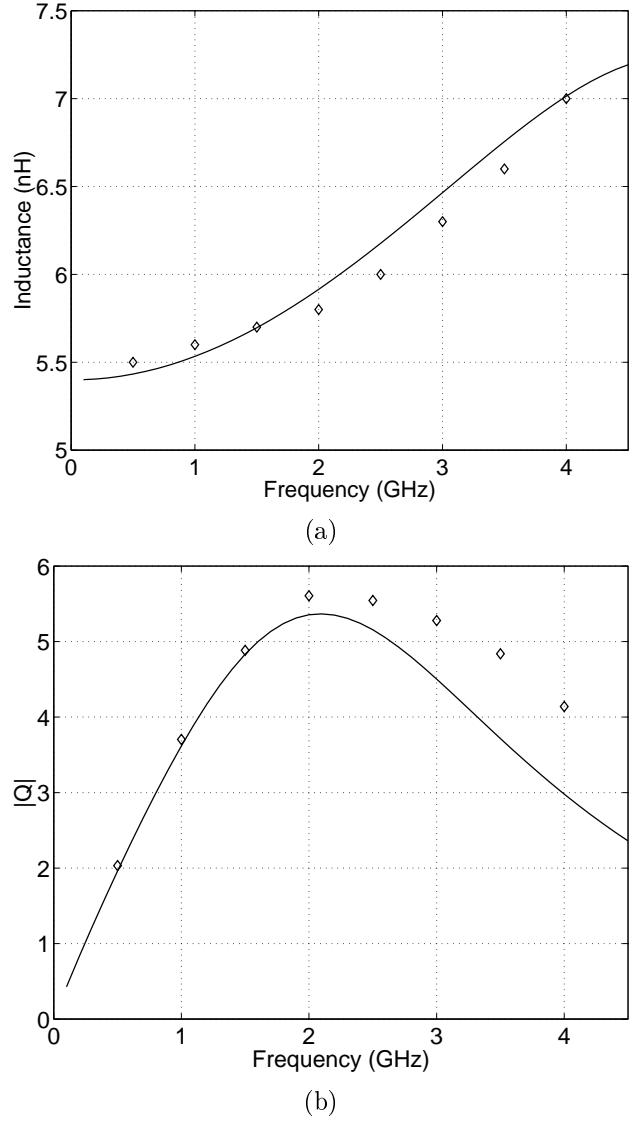


Fig. 4. Comparison of (a) inductance and (b) quality factor for simulations without Si substrate effects (-) versus measured data (◊) [9] for a 5 nH spiral with  $\sigma_{Si} = 10 \text{ S/m}$

inclusion of the substrate skin effect is necessary for accurate modeling of the structure. In addition, it is observed that the ladder network approximation yields a response virtually identical to the distributed model for frequencies from dc to near the first resonance.

## VI. Conclusion

The frequency dependent behavior of microstrip in lossy media, such as in Si-SiO<sub>2</sub>-based struc-

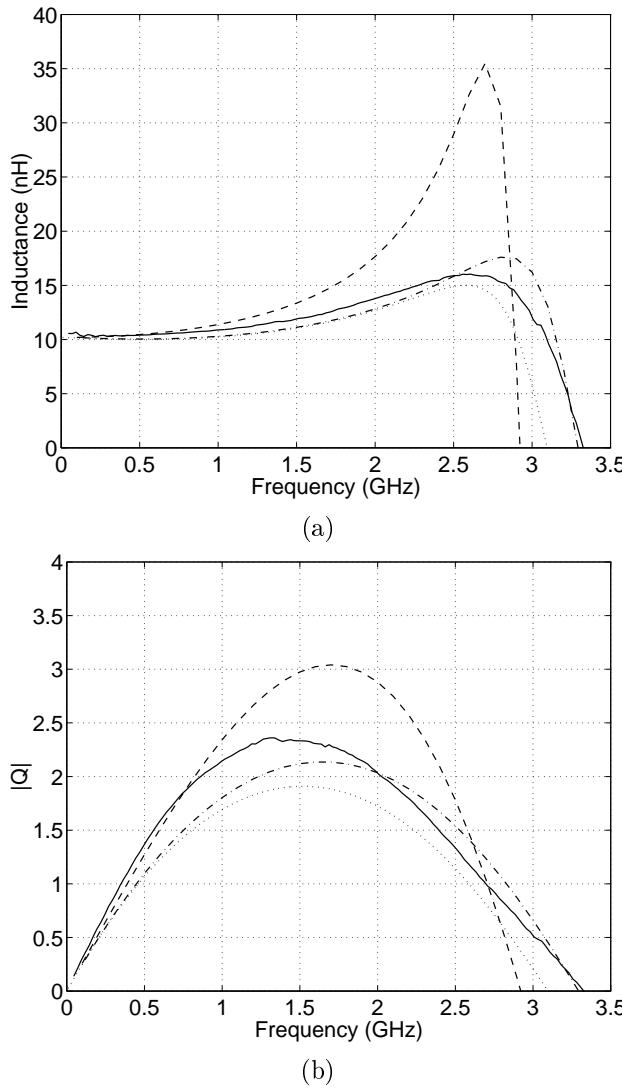


Fig. 5. Comparison of (a) inductance and (b) quality factor for simulations with (-. distributed, .. ladder) and without (- -) Si substrate effects, versus measured data (-)

tures, necessitates a model which can accurately account for all dominant substrate effects. The proposed model for spiral inductors on lossy substrates are in better agreement with measured data for higher conductivity-frequency products ( $\sigma\omega$ 's) than previous models which only address transverse currents, as demonstrated in the preceding section. In conclusion, it is seen that the skin-effect mode must be considered for accurate analysis and modeling of spiral inductors fabricated on lossy substrates as in CMOS technology.

## Acknowledgement

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